FOLIA 273

Annales Universitatis Paedagogicae Cracoviensis

Studia ad Didacticam Mathematicae Pertinentia 10(2018)

ISSN 2080-9751
DOI 10.24917/20809751.10.3

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A case study on the semiotic signs during a lecture of Cartesian plane*

Abstract. The use of signs in the teaching of mathematics plays a crucial role in students' cognition. Trying to understand what signs both students and teachers use in the mathematics classes may help us understand their meaning making processes. From this point of view, this paper aims to reveal the use of semiotic resources both by students and a teacher while lecturing Cartesian plane in seventh grade level. The study group of this qualitative study consisted of a teacher and her 29 seventh grade students. Two lectures delivered by the teacher on the Cartesian plane have been video-recorded. The semiotic analysis to make meanings from the linguistic and visual signs expressed through gestures and discourses has been conducted. According to the analysis of the data, links were explored between the signs in the class to present what the students and teacher actually were trying to say and how the signs used were reflected on the other side. What is more, the unclear directions and the inconsistencies between the discourses and gestures of the teacher misled the students' thinking, which was revealed during the mathematical tasks. It seems that the teacher assumed that students think the same way with her.

1. Introdution

Communication refers to the act of sending and receiving messages, which can be considered as the system of signs. It can be achieved through a number of forms. While language is a powerful model for understanding communication, it has limitations such as an explanatory schema because it ignores other forms of communication (e.g. visual or gestural) (Moriarty, 1996). Humans can communicate and convey meanings through their actions, which can be thought as the collection of signs. Hence, we have to try to understand what these signs mean to

*2010 Mathematics Subject Classification: Primary: 97C30; Secondary: 97D40, 97E40
Key words and phrases: : semiotics, sign, mathematics, Cartesian plane, ordered pair

really understand what people actually say while communicating. Since, it can be claimed that all that we can know/perceive is mediated by signs.

Semiotics can be defined as the study of signs and symbols as elements of communicative behavior (Random House Electronic Dictionary, 2017) or most commonly as a science that studies the life of signs within society (Saussure, 1916/1983). Hence, it can be defined as the study of signs and their meanings in short. It deals with how meaning is constructed and understood. Studying with signs allows us to uncover meaning and to see what is hidden behind the curtains. A sign can be a word, a sound, a gesture, an object, or a visual image. As Deely (1990) puts forth, ”at the heart of semiotics is the realization that the whole of human experience, without exception, is an interpretive structure mediated and sustained by signs” (p.5). We make meanings through our creation and interpretation of signs. A number of people have studied the discipline of semiotics for the late centuries. But two important figures in the development of semiotics could be Charles Sanders Peirce [1839–1914] on its philosophical side and Ferdinand de Saussure [1857–1913] on the linguistic side (Nöth, 1990).

Saussure defines sign as a two-fold entity consisting of the signifier and signified (1916/1983). The signifier is defined as the physical part of the sign, the actual substance of which it is composed (sound waves, alphabet characters etc.). The signified however is defined as the meaning or mental concept to which the signifier refers (Danesi, 1994). On the other hand, Pierce considers the sign as a composition of the representamen or sign, the object or that to which the representamen refers, and the interpretant or the individual's comprehension of the representamen/object association. The representamen is synonymous with Saussure's signifier, while the object and interpretant are Saussure's signified in two parts. Hence, it can be said that the dyadic model of Saussure becomes triadic in Piercian view of semiotics. Pierce also claimed the existence of three relations between signifiers and signified. Pierce categorized the patterns of meaning in signs as iconic (resembling or imitating the signified), symbolic (needs to be learned/arbitrary), and indexical (existential connection to the signified/cause) (Houser, Kloesel, 1992).

Regarding this study, we also want to mention Barthes's theory of semiotics shortly. Barthes (1964/1983) introduces two concepts within semiotic reading: Denotation, which is the type of lexical meaning associated mainly with arbitrary signs (what we actually see / the surface meaning) and connotation, which is nonlexical type of a meaning mainly associated with indices and relies on one or more previous levels of semiosis (what you associate with an image or any sign / the deeper or hidden meanings and associations). When talking about mathematical world, symbols convey or substitute conceptual entities (meanings). However, the crucial point in mathematics instruction processes is not just mastering the syntax of the mathematical symbolic language, but mastering its semantics. That is to say, understanding the nature of mathematical concepts and propositions and their relationship to contexts and problem-situations (Godino, Batanero, 2003).

Eco argues that a sign is not only something that stands for something else, but must also be interpreted (as cited in Lechte, 2001). One way to understand how interpretation works is to analyze the logical process, by which we create inferences and make sense of things (Langrerhr, 2003). Our cognitive functioning is intimately linked and affected by the use of signs (Radford, 2000). According to Eco and Sebeok (in Langrehr, 2003), the message recipient is seen as an investigator or message detective who is trying to reach a conclusion by making educated guesses about seemingly ambiguous information. We learn to read the world in terms of the codes and conventions which are dominant within the specific sociocultural contexts and roles within which we are socialized (Chandler, 2017). People use signs to construct meaning mediated by personal experience and culture. We can say that there is no universally acceptable language of visual description. If we think of every class as a small community, then each of them should possess its own unique culture, in which there are many signs specific to the existing environment including students and teachers. Furthermore, every course may have its own culture and signs.

A growing number of researchers in mathematics education have been applying the tools of semiotics to mathematics education especially since the late 1990s. Among those researchers, Radford (e.g. 2000, 2003) conducted studies on generalization of algebraic patterns and analyzed the mathematical thinking processes of students in terms of semiotics. Ball (2011) also studied about the signification processes of students for the understanding the concepts of rate and ratio. Chapman (1995) focused on the use of language in a mathematics classroom in order to see the role of language in constructing the shared meanings of a mathematical theme. There are a number of other studies focusing on the role of semiotics in mathematics education (e.g. Arcavi,2003; Ernest, 2006; Morgan, 2006; Pimm, 1995). However, a study on the use of semiotics in teaching Cartesian plane has not been met in the literature. Since, it is essential for the learning of analytical geometry, we tried to examine an example of instruction on Cartesian plane in terms of semiotic perspective.

The use of signs in the teaching of mathematics plays a crucial role in students' cognition. Trying to understand what signs both students and teachers use in the mathematics classes may help us understand their meaning making processes. The semiotic signs can be considered as the way of expressing their thinking processes, as they are the mediators of the sociocultural participation. That is why we have to take into consideration all the semiotic signs taking place in the classroom teaching and interactions, since they help us understand what is going on within the minds of the individuals, especially students. From this point of view, this paper aims to reveal the use of semiotic resources both by students and a teacher while lecturing Cartesian plane in seventh grade level.

2. Methodology

The study group of this qualitative case study consisted of a teacher and her 29 seventh grade students. The teacher has nine years of experience and has been working with the participating students for the last two years. Two lectures delivered by the teacher on the Cartesian plane have been video-recorded. The videos included the in-class interactions and dialogues between the teacher and students as well as the ones among the students. The meanings of the semiotic signs used have been explored through an investigation of the actions took place in the class. The semiotic analysis to make meanings from the linguistic and visual signs expressed through gestures and discourses has been conducted. In an effort to avoid the misleading assumptions, the both researchers argued about the inferences made to come up with a better decision reducing the researcher's bias. The meaning making processes in the classroom have been analyzed through a social constructionist perspective.

3. Findings

In this section, we presented a picture of the mathematics courses to give an impression of the context. Besides, the semiotic resources have been revealed.

The teacher introduced the coordinate plane exemplifying with map, plane and ship to determine the location. The last two examples were the chairs at the cinema (e.g. D5) and the positions of the students in the classroom (assuming the ground as the plane) to let them have the idea of point. The teacher wanted to make sure that the students all understand the same thing. Later she introduced the x- and y- axes with horizontal and vertical lines during the transition to the mathematical world. She labeled the intersection as the starting point (0) and numbered the x-axis first to the right (1, 2, 3, 4, 5, ...) and to the left (-1, -2, -3, -4, -5, ...). The discourse of the teacher here had a function of introducing new concept with the use of previously known facts by students. Also her use of gestures for the horizontal (↔) and vertical (↕) lines helped students to visualize the axes. She tried to make students remember the number line and relate with the x-axis. For the y-axis she said:

> Teacher (T) : Let's think this as the straight number line! (showing vertically). Or we can think of this as the floors of an apartment. We enter the building from this point (0), if I go upwards 1, 2, 3, 4, 5,... and if I go downwards -1, -2 ,-3, -4, -5, ...

The teacher assumed that all the students would realize the same thing from what she meant. In this situation, she probably meant to use the logic of an elevator for going up and down on the vertical line. In this context, this may be thought of an example of what Barthes (1964/1983) conceptualizes as connotation. However, some]of the students thought differently. Probably she was expecting this to be a good analogy for the students until a student asked:

> Student 1 (S1): Then do the sides represent the number of rooms? (pointing his finger from left to right)

She slid over this question and avoided putting emphasis on this controversial issue contenting with saying that "We can't think like that". She named the regions anticlockwise using a gesture circularly starting from the first region and explained:

> T: We try to find the points here [as in the cinema]. We have called the intersection as "0" and this midpoint is called as origin.

Then she changed what she said:

> T: The name of this point is 0 and 0.

She labeled (0,0) as the name of that point without explaining why it includes two-0s. Probably she was unaware the fact that first she named the midpoint as ”0”, then as ”0 and 0”. She said that with a routine and did not think that the students were new to this kind of labeling. In fact, this might have been a good introduction for labeling the points on the coordinate plane. She had mostly used informal language till now and used her hand effectively to show what she was saying. She had an explanation and tried to relieve the students:

> T: We will now use two foreign names (implying abscissa & ordinate) but you will get used to it.

She used another example of battleship game to make students have a better idea for the coordinate plane. This may be thought to be helpful for students' understanding and visualization. She gave the formal definition of coordinate plane emphasizing the verticality and the number lines for both axes: ”Vertical intersection of two number lines at the point 0”. This definition might also be used as a chance to introduce why the origin labeled as (0, 0). This would also open window for labeling the other points. She also explained (a,b) as the ordered pair showing the location of a point, just as C3 at the cinema.

She determined a point A and called it as the point (3, 2) showing the numbers on the axes.

> T: While we use C3 at the cinema, we will use two numbers here. The first digit comes from x-axis, second one comes from y-axis. Where does A correspond to for x-axis? y-axis? (Moving her hand from the point to the x- and y- axes respectively, vertically and horizontally) We name the number on the x-axis as the abscissa and the number on the y-axis as the ordinate.

She made students draw coordinate plane on their notebooks limiting them for numbering between -5 and 5, just as she introduced at the beginning of the lesson.

> T: 5 is enough to go, don't need to write more.

She justified herself saying that the largest number for the coordinates is 5, but we are not sure that the students really got the idea. After a student drew the lines till 5 and -5, the teacher extended the line saying that they can draw a little bit further (probably just to show the arrows). She further instructed the students:

> T: Writing x and y next to the axes is compulsory!

The students compared their work as a part of the classroom norms.

A student drew the coordinate plane as below labeling the origin as 1.

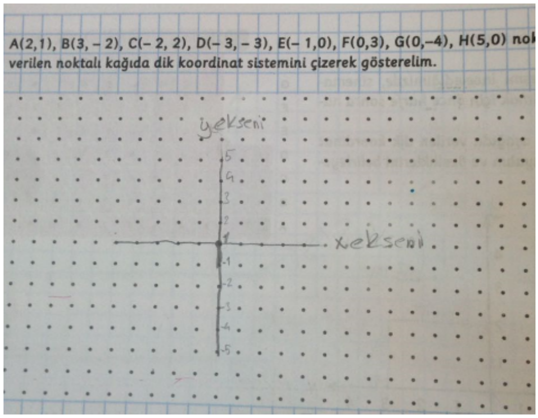

Fig. 1. Labeling the origin as 1

She directly explained the mistakes of the students without asking the others or the student himself/herself. Another student numbered the line from up to down 1 to 8. The teacher showed the student's work on the board:

> T: Where to write 9? No place? Do you enter the building using the 8th floor? (insisting her analogy although she must have understood the risk of it) We enter from 0th and go upwards as 1, 1, 2, 3, 4. Attention to this. This point is (0,0) (showing the origin) and, in fact, all the numbers scatter from this point (showing origin). (Marking the beginning points of the line segments starting from positive y-axis) 1, 1, 1, 1.

She noticed the mistake in a few seconds and corrected what she said and wrote as 1, -1, -1, 1. She completed the number lines with numbers on them explaining again how it is constructed (one more, one more, one more,. . .). In fact, she could have noticed the mistake of her analogy for scattering.

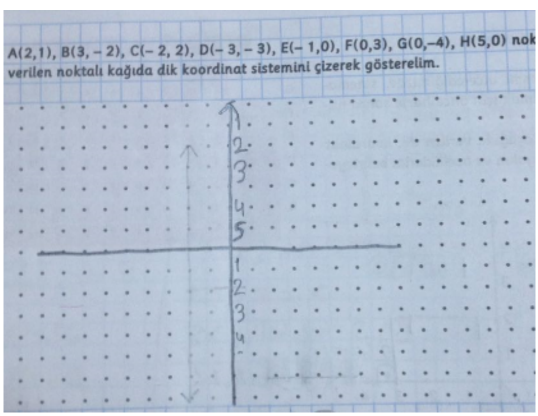

Fig. 2. Mislabeling positive y-axis

> T: Write the numbers till the arrow symbol. (She again wrote until 5.) Where to write 6? Erase the arrow and write down 6. We start from the origin not

> from the arrow. Write down the names of the axes: x to the right and y to the top. Let's show A(2,1). Where should we take the first? For the alphabet, we know it goes on like x, y, z... (She again tries to make students get the idea through another fact which she thinks as easy to remember) So, first we take from x, then from y. Then we should take 2 from x and 1 from y, and bring together. You have to sit down this chair. We align little by little like this (marking the dots showing the alignment from the axes).

She emphasized that these alignments are not straight lines, since it can be confused with the axes.

> T: After aligning 2 and 1 from the axes, then we need to intersect these. This intersection point is A.

The first student aligned the point from the x- and y-axes, respectively, starting from the axes just as the teacher did.

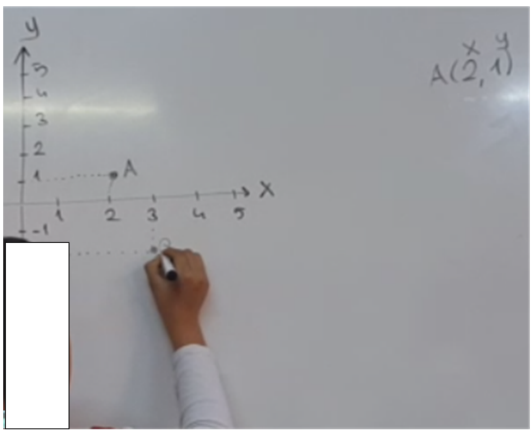

Fig. 3. Aligning from the axes

The next student was on the board to show (-2,2). The student showed -2 on the y-axis and looked at the teacher. Then the teacher asked:

> T: Where do we mark first?
>
> S2: From x-axis. (moving the board-marker to the -2 on x-axis.) We will start from here (-2 on the x-axis) and go to that point (2 on the y-axis).

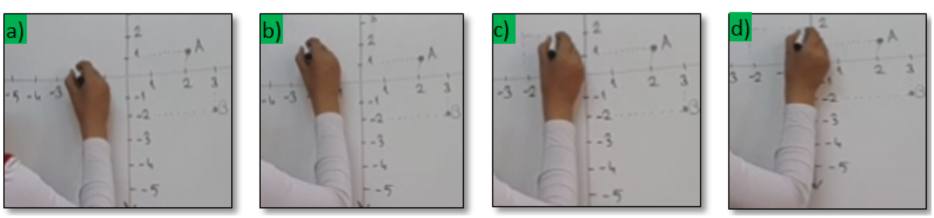

Fig. 4. Aligning from x-axis to y-axis directly

The student started marking from -2 on the x-axis (Fig.4a), went upwards (Fig. 4b), went to the right for 2 on y-axis (Fig. 4c and 4d). In fact, this student labeled the ordered pair correctly. After watching the video a few times, although the student was right, we noticed that this kind of labeling was a potential risk for the next students who followed this student's work. Next student correctly labeled point (-3,-3) starting from x-axis, going downwards, and then horizontally to the y-axis. However this method did not work for the next student. She had difficulty for determining the point (-1,0). When she came in front of the word, she showed -1 on the x-axis and moved to 0 at the origin in the air, trying to show how she would proceed. But when she moved upwards from -1 and turned right side to the y-axis, she noticed that her work was not like other students'. She was doubtful about her work and looked at the teacher and the coordinate plane waveringly. Since she stopped while moving to the right side to 1on y-axis, the teacher thought that the student meant the point (-1,1). The teacher explained that this point was called as (-1,1), hence it was not the right point. She again warned the students that they did not have to show these dashed lines to show the point correctly.

> T: Why do we do like that? To find the alignment of the point to the axes. I need the point. That is the important part for me, not the dashes. You will find the point (-1,0).

The student marks -1 and 0 (Fig.5a) respectively, as the teacher and other students did during their work. Then she started from -1 (Fig.5b), went upwards (Fig.5c), turned right towards y-axis (Fig.5d), and then went downwards to 0 (Fig.5e).

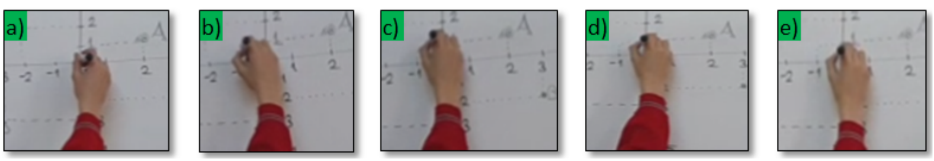

Fig. 5. False imitation to align from x-axis to y-axis directly

The teacher also came to the board and showed the students work to other students (Fig.6).

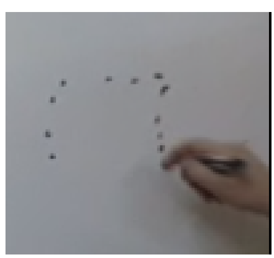

Fig. 6. Student's work in teacher's eye

> T: We never did something like that, didn't we? There is something missing somewhere. Why did you go to 1 on the y-axis and go downwards? Why didn't you go to 5 on the y-axis and go downwards?

> S3: I can't go straight ahead. 0 is in alignment with -1. (showing with the board-marker horizontally)

Teacher again reminded that they do not need the path. The point itself was the important one. The student marked the -1 on the x-axis as the point and teacher accepted it as the right answer. However, it was not obvious that the student marked that point knowingly. The teacher explained the correct answer once more, once she saw that the student had a difficulty in finding the point. She improperly emphasized that 0 meant ”absence”.

> T: Our point is (-1,0). What will I look for on x-axis? (Writing x on top of -1). What will I look for on y-axis? (Writing y on top of 0). That is to say do not look for anything on y-axis. There is no y component. y is 0. Only x is -1. -1 on x-axis is here. y is 0 here. we have to intersect these two. (Probably she understood that this will confuse the students' minds). Or think of that there is no y. Then out point is exactly here. (marking -1 on the x-axis.)

This kind of guidance could make students memorize this and focus just the other number in the ordered pair.

4. Conclusion

We aimed to search for meaning in gestures and discourses of teacher and students, which was about reading between their words and moves. The findings revealed that the semiotic signs created by speeches and gestures of the students and the teacher support and interplay each other. Verbal and visual signs should be used in concordance with each other and carefully, since both kinds of signs affect the students' interpretation. Links were explored between the signs in the class to present what the students and teacher actually were trying to say and how the signs used were reflected on the other side. What is more, the unclear directions and the inconsistencies between the discourses and gestures of the teacher misled the students' thinking, which was revealed during the mathematical tasks. It seems that the teacher assumed that students think the same way with her.

This problem showed itself in the semiotic signs used by a student trying to show a point on the x-axis, after the teacher showed how to mark a point on the coordinate plane using the alignments from the axes pointing with her hands both vertically and horizontally. This example reveals two important points for educators: First, sometimes the students learn from their friends and imitate their peers rather than the teacher. Second, and more importantly, the ways of students' reasoning, whether right or wrong, show themselves through their discourses and gestures, which helps us understand what is going on within their minds. Hence, teachers have to give the students chance to express themselves comfortably and encourage them to share how they think through discourses or gestures. Probably the students are unaware of the potential of their and the teacher's gestures, but the teachers should know the importance of the signs used within the classroom and try to learn how to rigorously read these signs, since these are the reflections of their ways of thinking and reasoning. Learning to read the signs used by students may help teachers to organize their methods of instruction and may structure their ways of teaching. Vygotsky (1997) claims that the gesture is the writings in the air. Furthermore, representations, discourses, gestures or any other signs can be perceived in different ways by different students. Berger (1984) states that the problem of meaning arises from the fact that the relation between the signifier and the signified is arbitrary and conventional; they can mean different things to different people. Hence it can be claimed that what is an icon or index to teachers may be perceived as a symbol or index to students. Accordingly, the teachers should also be aware of different potential interpretations of signs. Hence, teachers can be careful about the use of signs during the instruction and they can focus on the signs used by the students in addition to the discourses. Furthermore, students tend to imitate/copy what the teacher does mostly without understanding. This also shows the importance of the signs used by the teacher and his/her way of instruction. Radford (2003) also states the importance of semiotics in mathematics classrooms in the search of understanding of students' mathematization processes. Trying to solve the meanings of the gestures and other forms of signs in the classroom may let us analyze the mathematical teaching, learning and understanding processes deeply. Accordingly, it has been claimed that body movements and hand/arm-produced gesticulations manifest knowing (Presmeg et al., 2018), since embodiment perspectives emphasizes the role of the body moves in human communication (Proulx, 2013), which may be thought to signify the importance of semiotics for the learning and teaching of mathematics. To sum up, the teachers are suggested to learn to read the signs within the classrooms with the help of semiotics so that they may be aware of consequences of their use of signs and they may know more about what students reflect about their understanding with the use of verbal and nonverbal signs.

References

Arcavi, A.: 2003, The role of visual representations in the learning of mathematics, *Educational Studies in Mathematics* **52**, 215–241.

Ball, T. S.: 2011, *Exploring the learning of ratio by a semiotic process in a combined fifth- and sixth-grade class* (Doctoral dissertation), University of Nevada, US.

Barthes, R.: 1964/1983, *Elements of Semiology* (Trans. A. Lavers & C. Smith), Hill and Wang.

Berger, A. A.: 1984, *Signs in contemporary culture: An Introduction to Semiotics*, Longman.

Chandler, D.: 2017, *Semiotics: The basics*, Routledge.

Chapman, A.: 1995, Intertextuality in school mathematics: The case of functions, *Linguistics and Education* **7**, 243–262.

Danesi, M.: 1994, *Messages and Meanings: An introduction to semiotics*, Canadian Scholar's Press Inc.

Deely, J.: 1990, *Basics of semiotics*, Indiana University Press.

Ernest, P.: 2006, A semiotic perpective of mathematical activity: The case of number, *Educational Sciences in Mathematics* **61**, 67–101.

Godino, J. D., Batanero, C.: 2003, Semiotic functions in teaching and learning mathematics, *Educational perspectives on mathematics as semiosis: From thinking to interpreting to knowing*, 149–167.

Houser, N., Kloesel, C.: 1992, *The essential Peirce: Selected philosophical writings*, Indiana University Press.

Langrehr, D.: 2003, *From a semiotic perspective: Inference formation and critical comprehension of television advertising*, retrieved from: `http://www.readingonline.org/articles/langrehr/`.

Lechte, J.: 2001, *Fifty key contemporary thinkers: From structuralism to postmodernity*, Routledge.

Morgan, C.: 2006, What does social semiotics have to offer mathematics education research?, *Educational studies in mathematics* **61**(1–2), 219–245.

Moriarty, S. E.: 1996, *Abduction and a theory of visual interpretation*, retrieved from: `http://www.spot.colorado.edu/~moriarts/abduction.html`.

Nöth, W.: 1990, *Handbook of semiotics*, Indiana University Press.

Núñez, R. E.: 2009, Gesture, inscriptions, and abstraction: The embodied nature of mathematics or why mathematics education shouldn?t leave the math untouched, w: W. M. Roth (red.), *Mathematical representation at the interface of body and culture*, Information Age Publishing, Charlotte, NC, 309–328.

Pimm, D.: 1995, *Symbols and meanings in school mathematics*, Routledge, London.

Presmeg, N., Radford, L., Roth, W. M., Kadunz, G.: 2018, *Signs of signification: Semiotics in mathematics education research*, Springer, Cham, Switzerland.

Radford, L.: 2000, Signs and meanings in students' emergent algebraic thinking: a semiotic analysis, *Educational Studies in Mathematics* **42**, 237–268.

Radford, L.: 2003, Gestures, speech, and the sprouting of signs: A semiotic-cultural approach to students' types of generalization, *Mathematical thinking and learning* **5**(1), 37–70.

Random House Electronic Dictionary.: 2017, *Random House Webster's Electronic Dictionary and Thesaurus.*

Saussure, F.: 1916/1983, *Course in general linguistics* (Trans. R. Harris), Duckworth.

Vygotsky, L. S.: 1997, *Collected works (Vol. 4)* (R. Rieber, Ed.), Plenum, New York.

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